Presented @ MENU 2010, Williamsburg VA, June 1, 2010

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Nucleon and Delta(1232) in chiral EFT

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Outline

- 1-slide introduction to chiral EFT
- Compton scattering off protons, polarizabilities
- Radiative pion photoproduction, Delta's MDM
- Chiral expansion in the complex plane

Chiral EFT = low-energy QCD

[Weinberg (1979), Gasser & Leutwyler (1984),
Gasser, Sainio & Svarc (1988), ...]

exploits the fact that the Goldstone bosons of spontaneous chiral symmetry breaking interact weakly at low energy (chi. sym. requires derivative couplings, and # of derivatives = power of momentum)

Lagrangian: $$\mathcal{L}(\pi, N, \ldots) = \sum_n c_n \mathcal{O}(p^n/\Lambda^n_{\chi SB})$$

S-matrix: $$S = \sum_n A_n(c_i)\,\frac{p^n}{\Lambda^n_{\chi SB}}$$

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S-matrix: $$S = \sum_n A_n(c_i)\,\frac{p^n}{\Lambda^n_{\chi SB}}$$

Near a resonance (or a bound state):

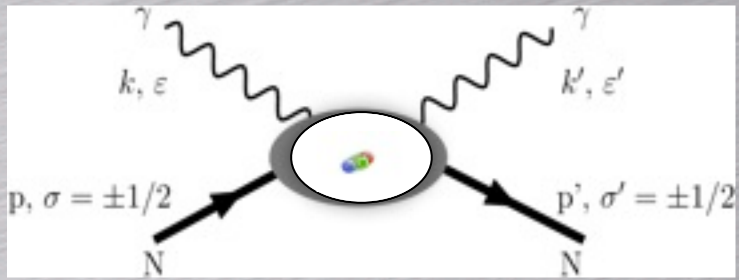
$$S \sim \frac{1}{s - M^2} = \sum_n A_n \left(\frac{p}{\Delta E}\right)^n$$

ΔE – excitation (binding) energy of the resonance (bound state)

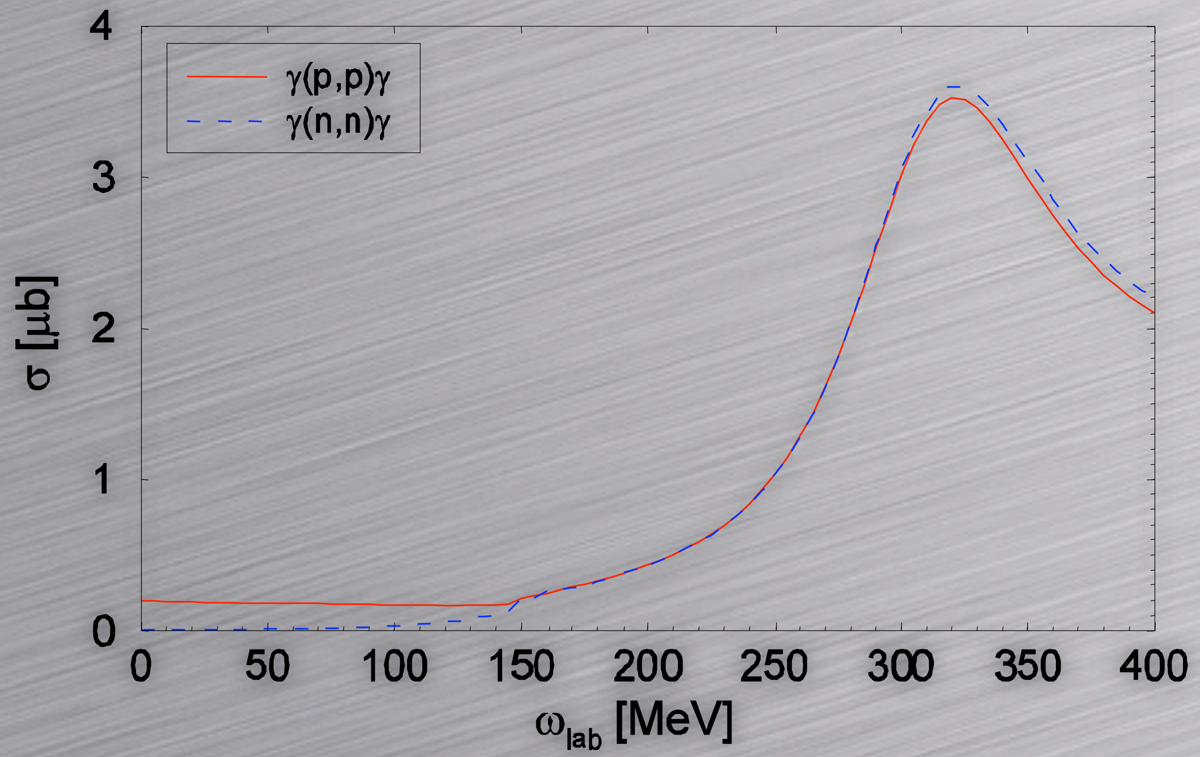
Chiral EFT with $\Delta(1232)$

$\Delta(1232)$ - first nucleon resonance, $\Delta E \equiv \Delta = M_\Delta - M_N \approx 300$ MeV

E.g., Compton scattering on the nucleon



Total cross-section at NLO
[V.P. & Phillips, PRC (2003)]

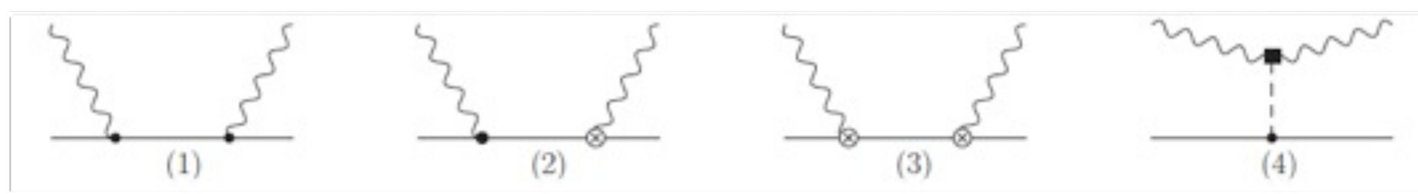


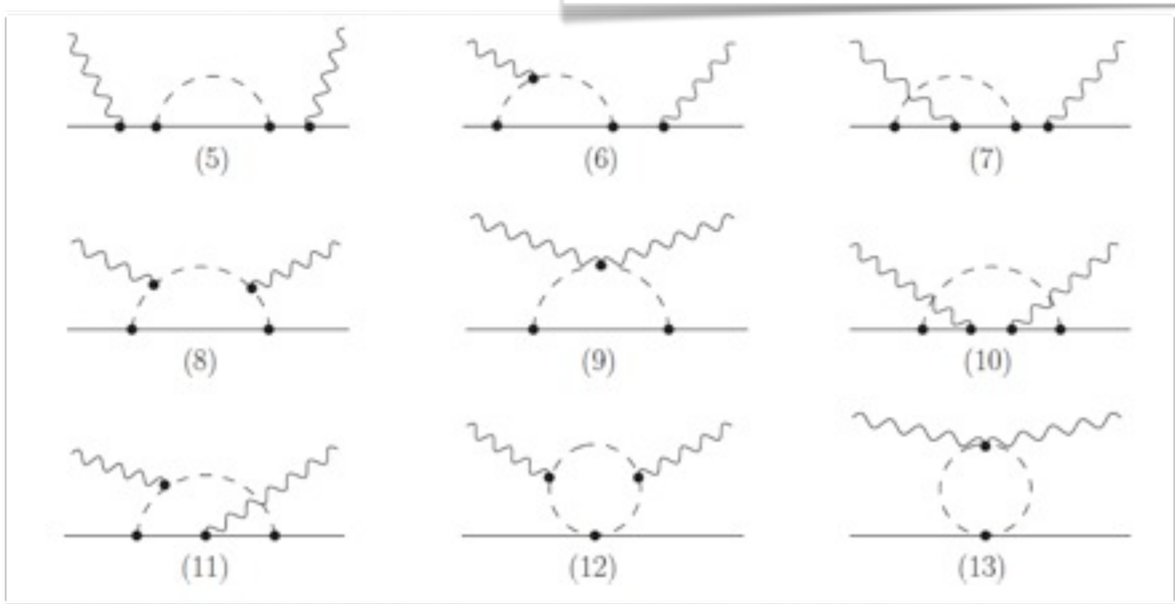
$$s = (p+k)^2 = M_N^2 + 2M_N\omega$$

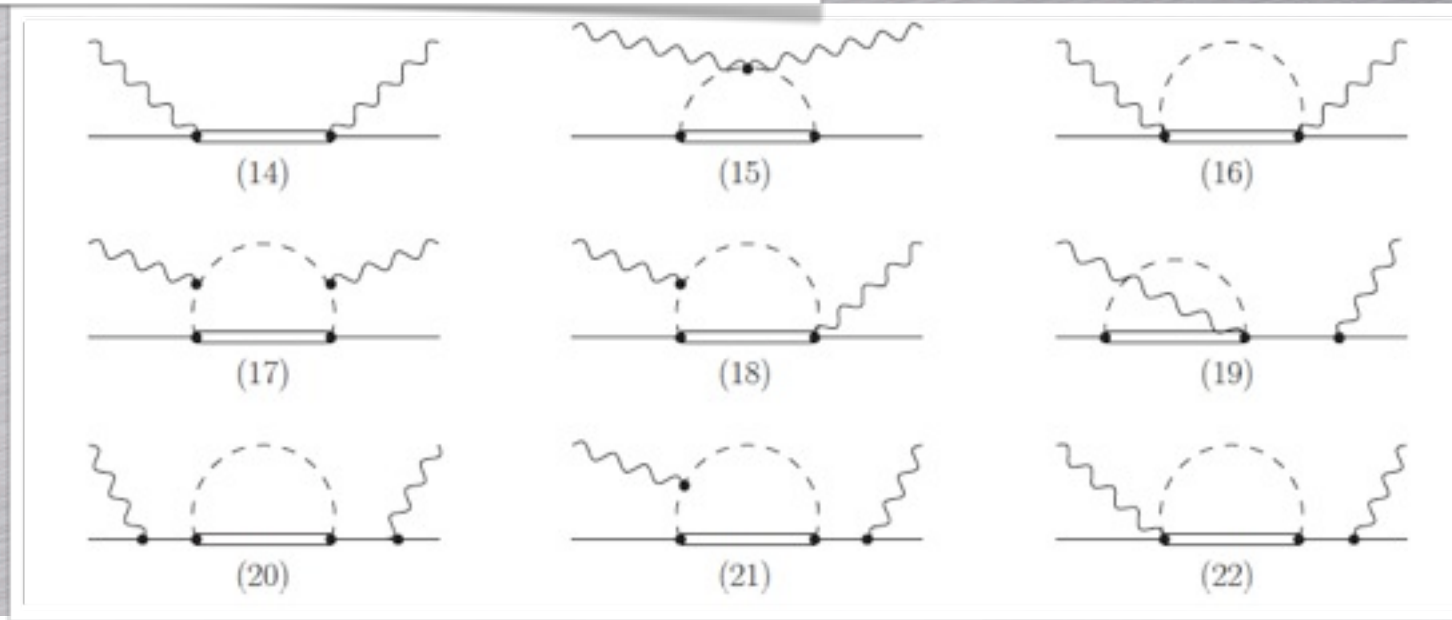
$$S \sim \frac{1}{s - M_\Delta^2 + iM_\Delta \Gamma_\Delta} \approx \frac{1}{2M_N}\frac{1}{\omega - \Delta + (i/2)\Gamma_\Delta},$$

$\Gamma_\Delta \simeq 115$ MeV – the width.

Compton scattering at NNLO
[V.Lensky & V.P., JETP Lett (2009); EPJ C (2010)]

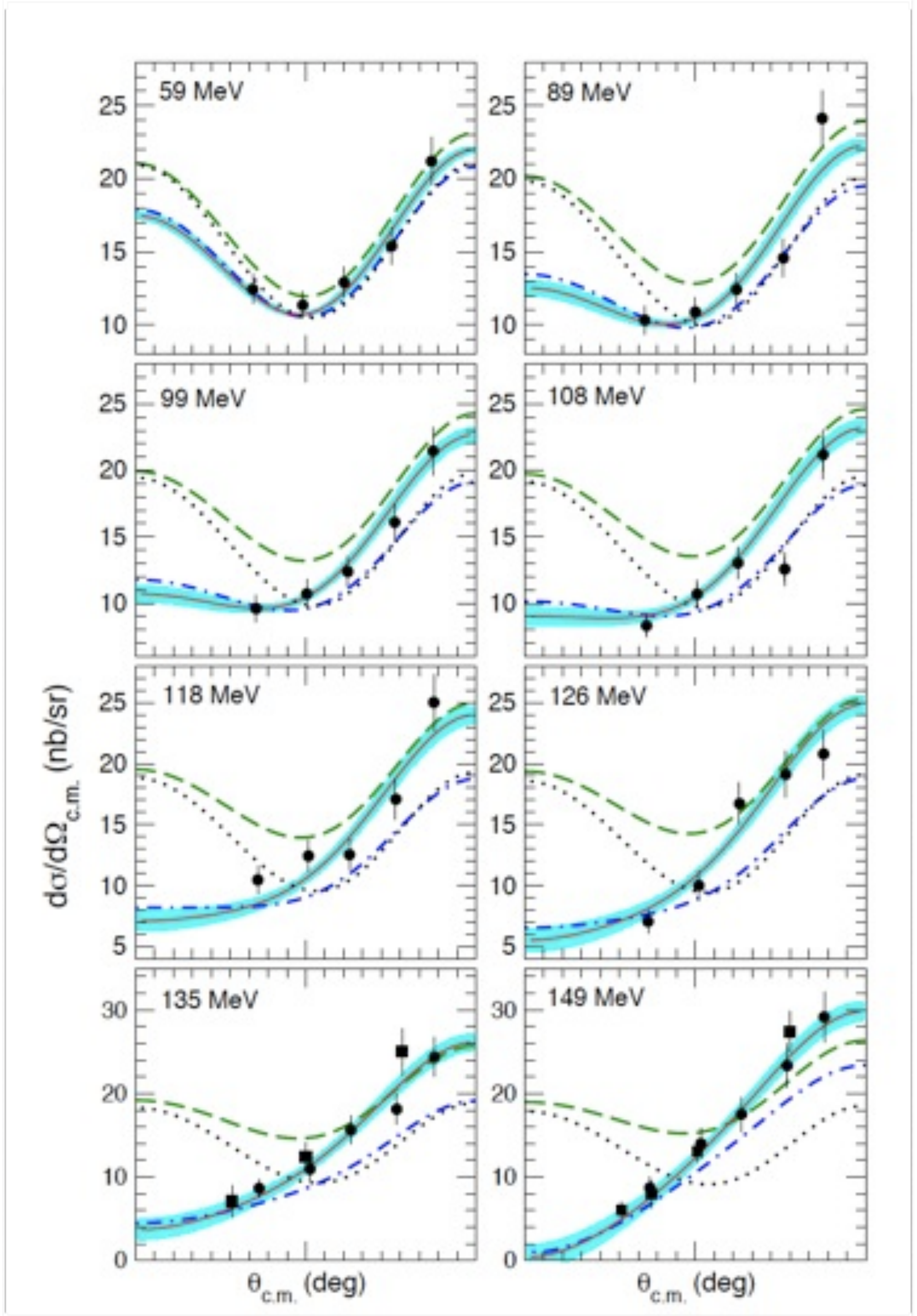






$\mathcal{O}(p^2)$	$\frac{e^2}{4\pi} = \frac{1}{137}$, $M_N = 938.3$ MeV, $\hbar c = 197$ MeV$\cdot$ fm
$\mathcal{O}(p^3)$	$g_A = 1.267$, $f_\pi = 92.4$ MeV, $m_\pi = 139$ MeV, $m_{\pi^0} = 136$ MeV, $\kappa_p = 1.79$
$\mathcal{O}(p^4/\Delta)$	$M_\Delta = 1232$ MeV, $h_A = 2.85$, $g_M = 2.97$, $g_E = -1.0$
$\mathcal{O}(p^4)$	$\alpha_0, \beta_0 = \pm\frac{e^2}{4\pi M_N^3}$

NNLO Compton cross sections and nucleon polarizabilities



Data points:
MAMI/TAPS (2001)
SAL (1993)
Illinois (1991)

Curves:

- Klein-Nishina
- Born + WZW
- + p-qube
- + Delta

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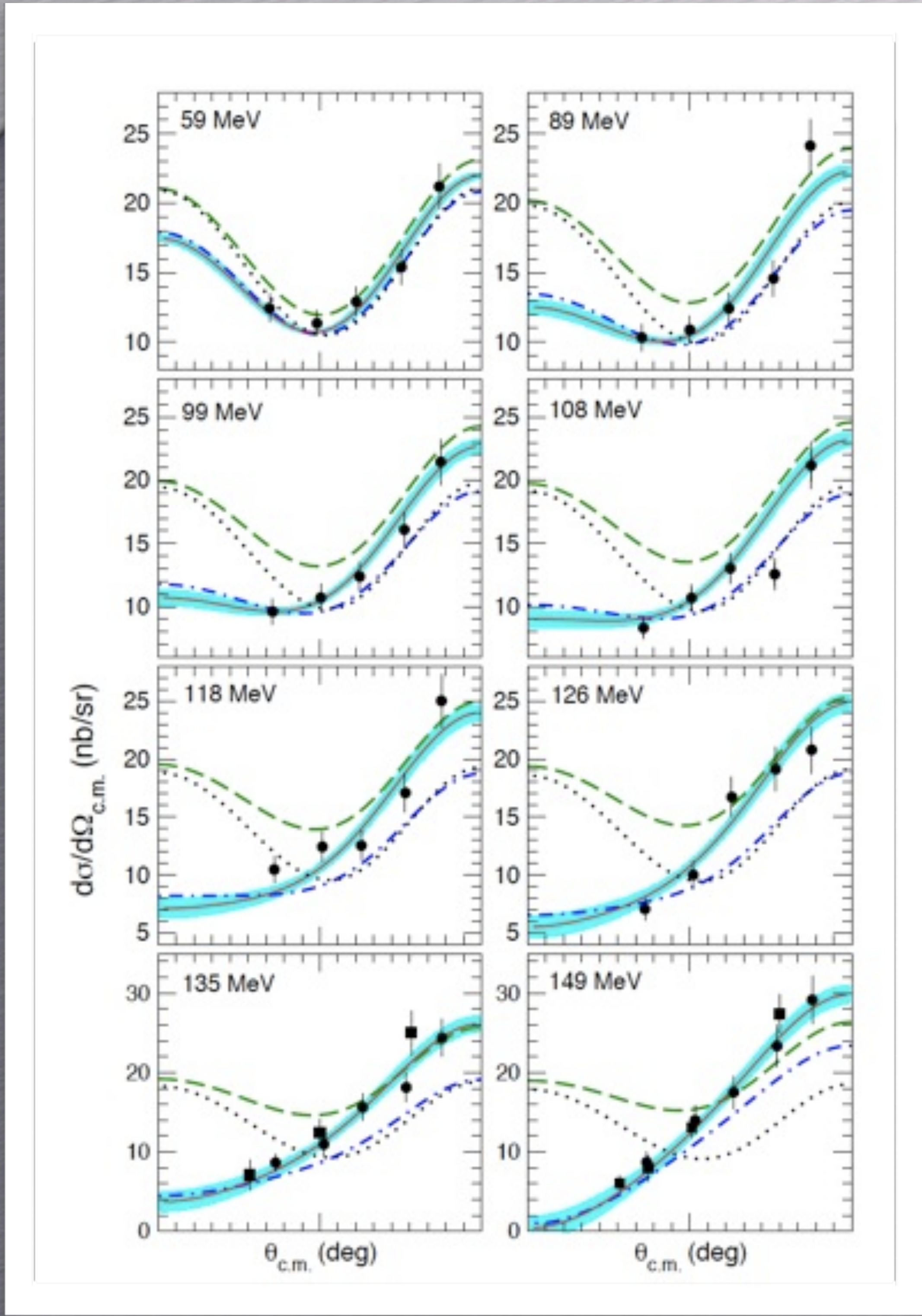
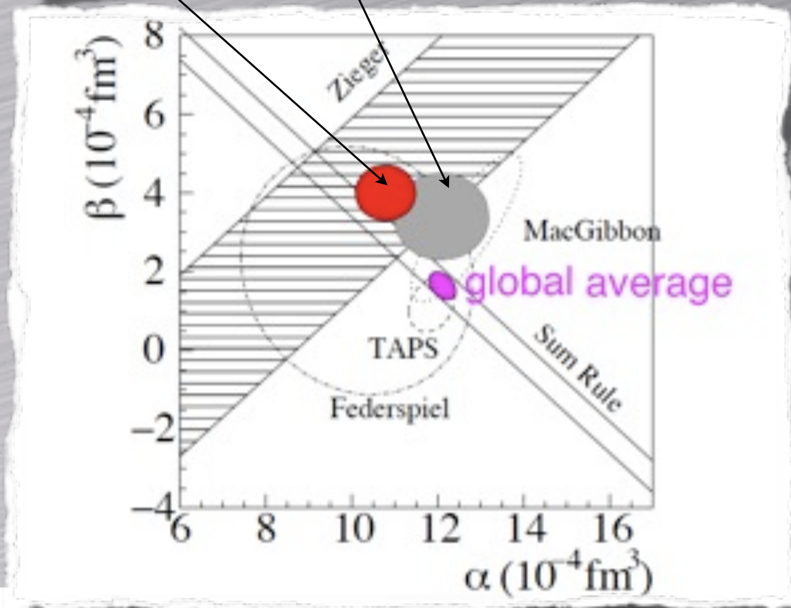


NNLO BChPT with Delta's

HBChPT $\mathcal{O}(p^4)$ [Beane et al (2005)]



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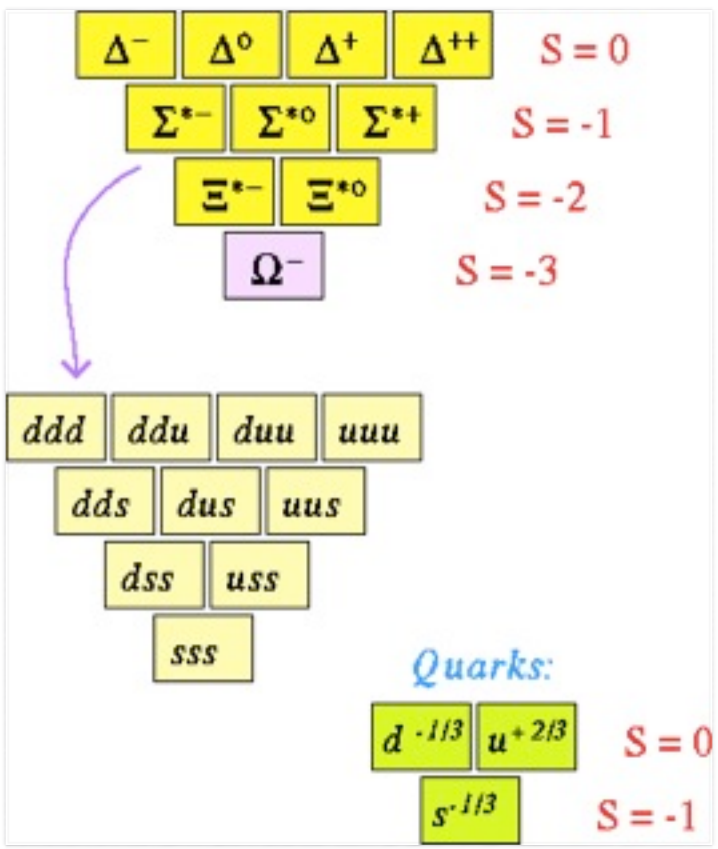
Problem: Cross-sections agree, polarizabilities do not

Delta-resonance as a particle with magnetic dipole moment

- Quark model, large Nc



In large-N_c limit:

- $M_\Delta - M_N = O(1/N_c)$
- $\pi N \Delta$ coupling, $h_A = \frac{3}{\sqrt{2}} g_A$
- $\gamma N \Delta$ coupling, $G_M = \frac{2\sqrt{2}}{3}\mu_p$, $G_E/G_M = O(1/N_c^2)$
- Magnetic moments, $\mu_\Delta = e_\Delta \mu_p$

- Natural value

Relativistic pointlike charge with mass M and spin s has magnetic dipole moment

$$\mu = 2s\frac{e}{2M}$$

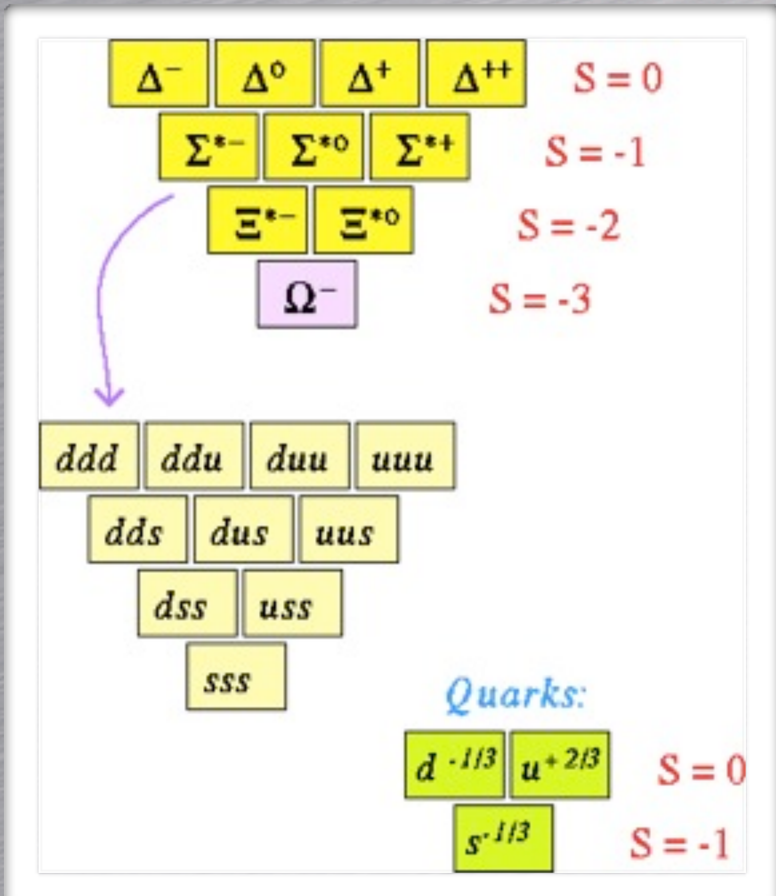
or, gyromagnetic ratio ***g*=2**

(GDH sum rule argument by S. Weinberg, 1972)

$$\mu_{\Delta^+} = 3\frac{e}{2M_\Delta} \simeq \frac{3M_N}{2.79\, M_\Delta}\mu_p \approx 0.82\, \mu_p$$

Delta-resonance as a particle with magnetic dipole moment

- **Quark model, large Nc**



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Natural and quark-model value - close

New lattice QCD results 1: external field method

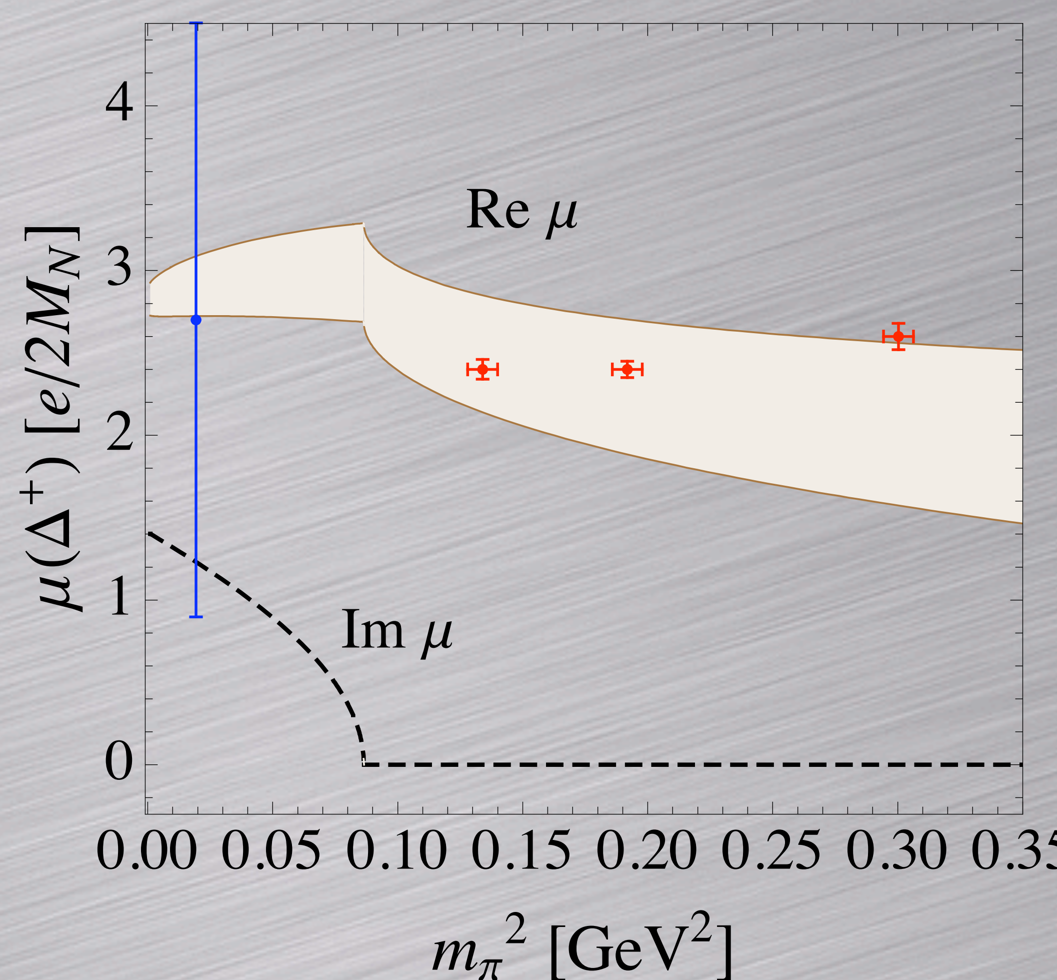
[Aubin, Orginos, VP, Vanderhaeghen, PRD (2009)]

m_π	$\mu_{\Delta^{++}}$	$\mu_{\Delta^{+}}$	$\mu_{\Delta^{0}}$	$\mu_{\Omega^{-}}$
548	3.65(13)	2.60(8)	-0.07(2)	
438	3.55(14)	2.40(5)	0.02(3)	
366	3.70(12)	2.40(6)	0.001(16)	−1.93(8)
PDG:	5.6(1.9)	2.7(3.5)	—	−2.02(5)

Data at phys. pion mass:
TAPS@MAMI Kotulla et al (2002)

Lattice data:
2+1 flavor clover QCD

Band: NLO ChEFT



New lattice results 2:
3-point function method

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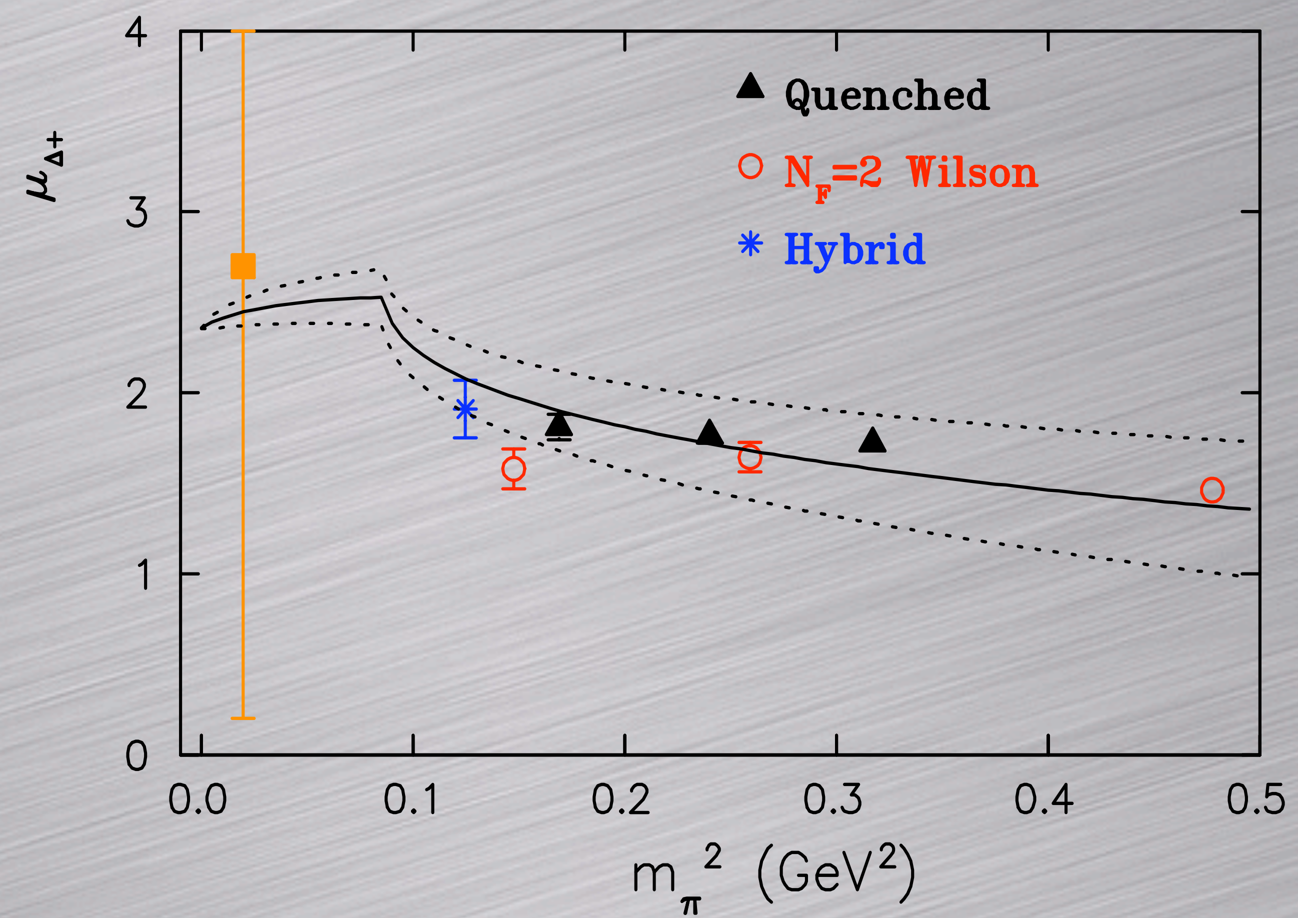

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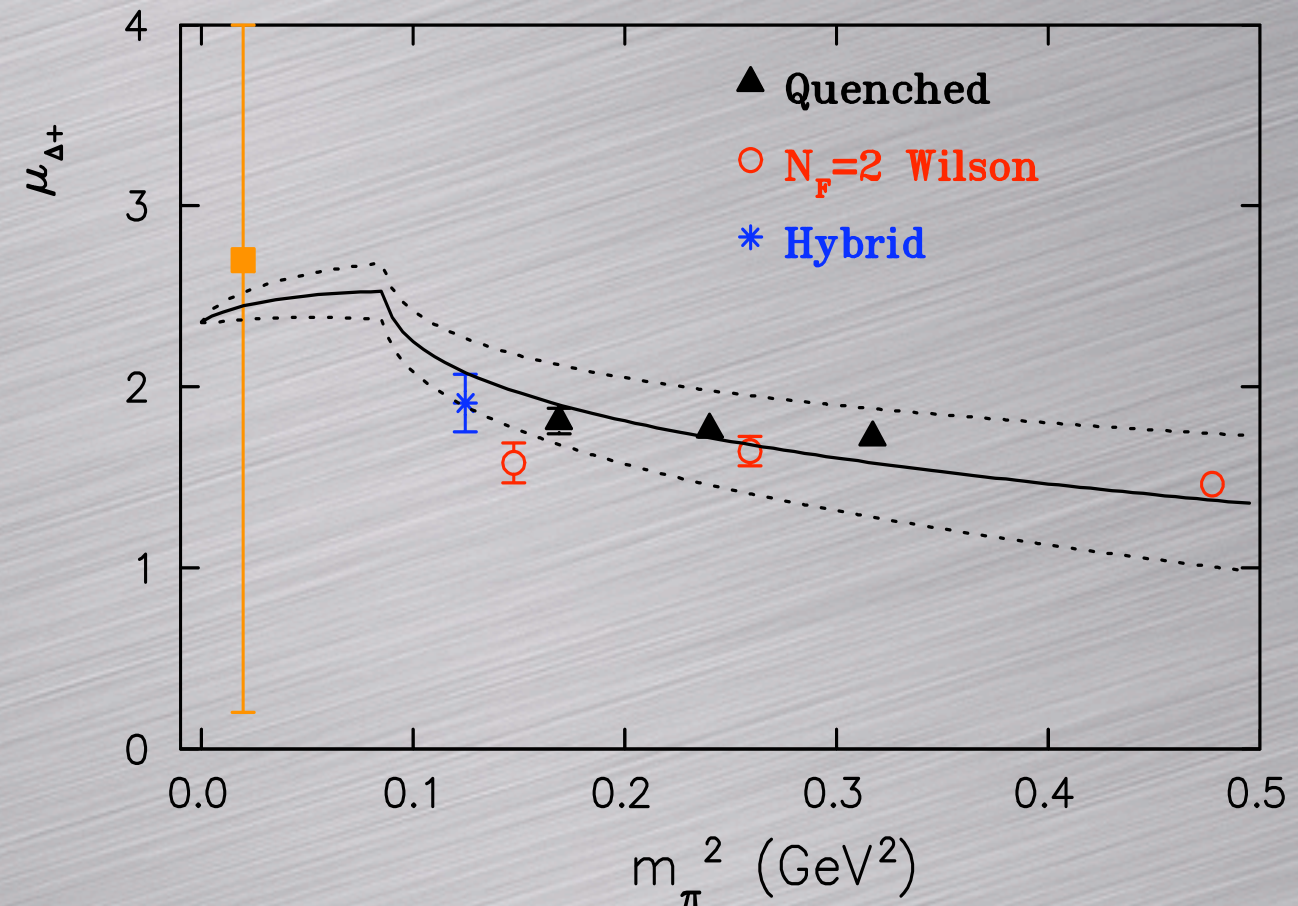

In present lattice calculations
Delta is stable!
$M_\Delta < M_N + m_\pi$

Defining and determining the Delta MDM

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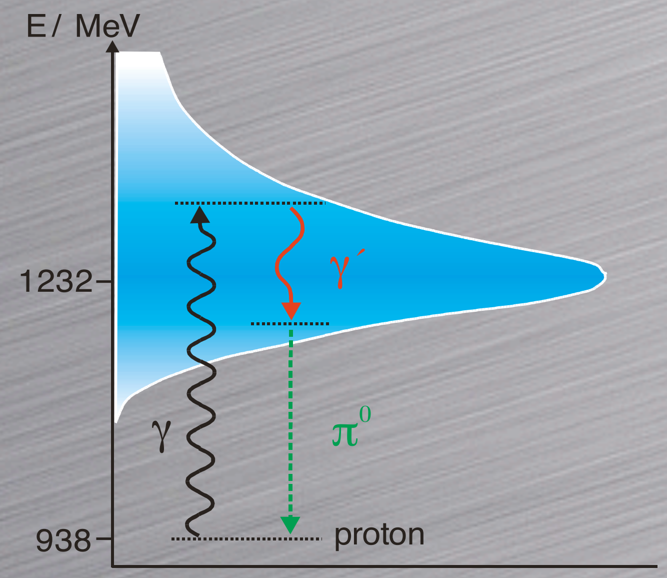
- Include as a field (Rarita-Schwinger vector-spinor) into the chiral effective Lagrangian.
- Power-counting for Delta contributions (SSE, ``delta-counting")
- Mass, MDM, etc., enter as parameters that need to be related to experimentally observable quantities (resonance position, width, etc.)

Radiative pion photoproduction



Machavariani, Faessler & Buchmann, NPA (1999), Erratum-ibid (2001).

Drechsel et al, PLB (2000)
Drechsel & Vanderhaeghen, PRC (2001)

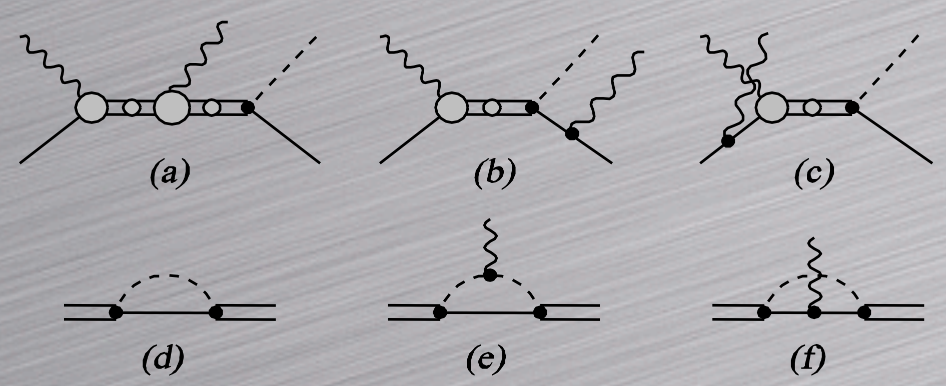
Chiang, Vanderhaeghen, Yang & Drechsel, PRC (2005).

Pilot experiment:
Kotulla et al
(TAPS@MAMI)
PRL 2002

V.P. & Vanderhaeghen, PRL (2005), PRD (2008)

Calculation to NLO in the δ expansion

2 free LECs – $\mu^{(s)}$ and $\mu^{(v)}$

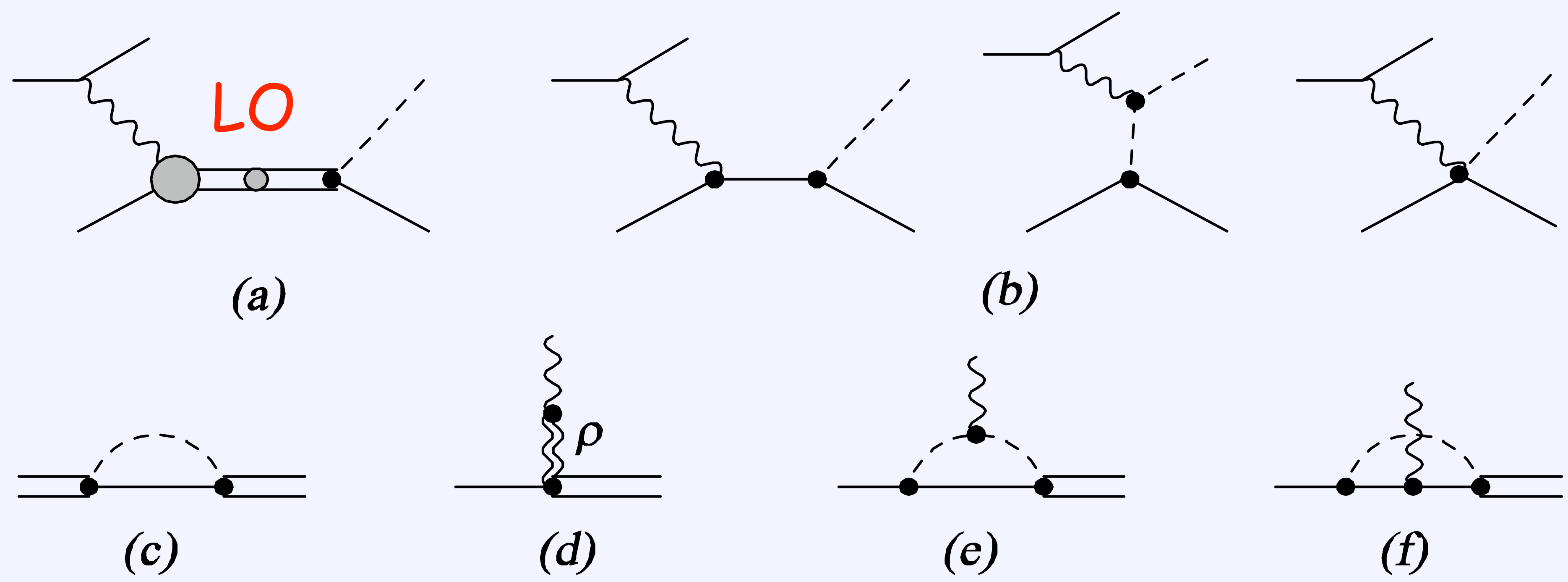


Dedicated experiment:
Schumann et al.
(CB&TAPS@MAMI-B),
EPJ A 2010

By-product experiment:
Prakhov et al.
(CB&TAPS@MAMI-C),
preliminary

Pion Electroproduction (eN->eNπ) in Δ(1232) region

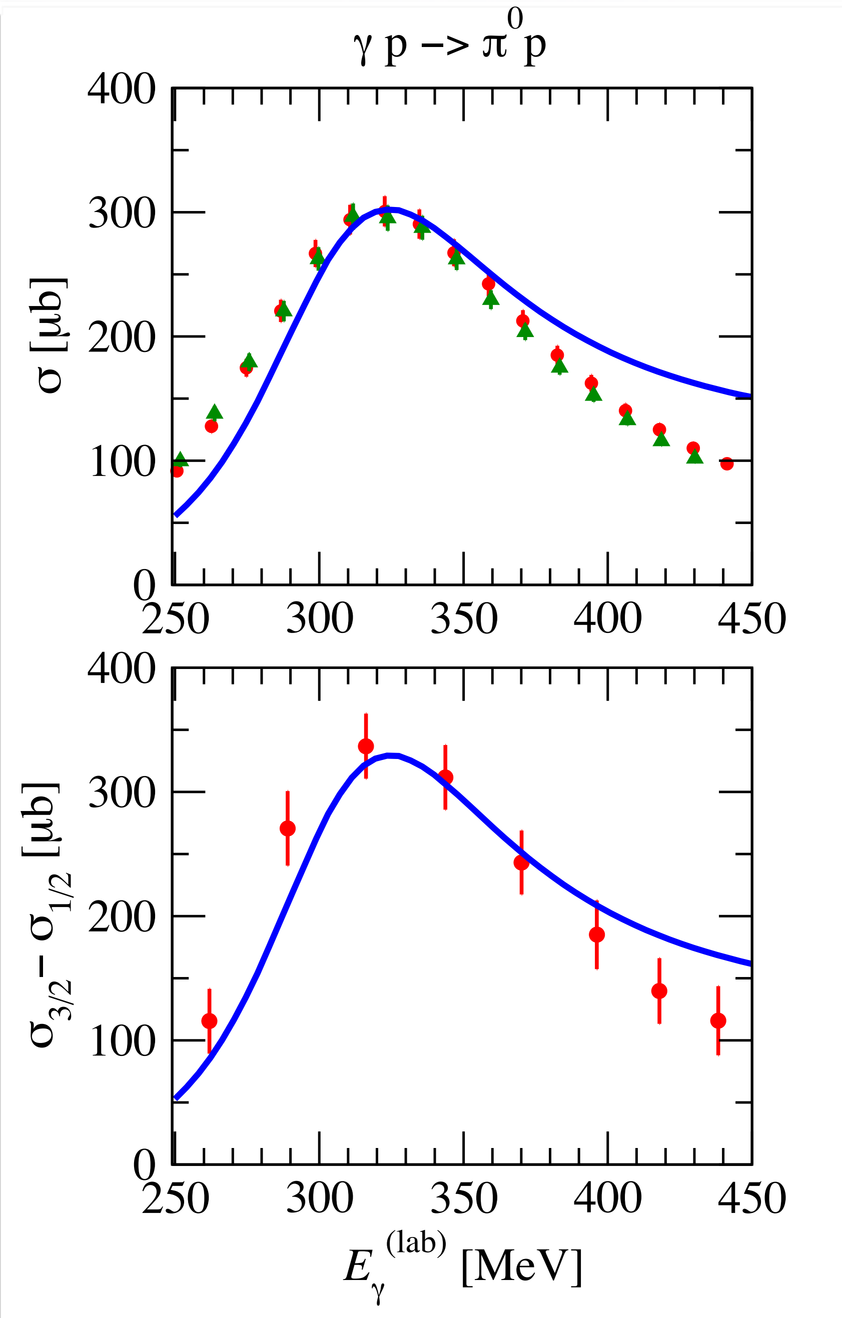
[V.P. & Vanderhaeghen, PRL 95 (2005); PRD 73 (2006)]

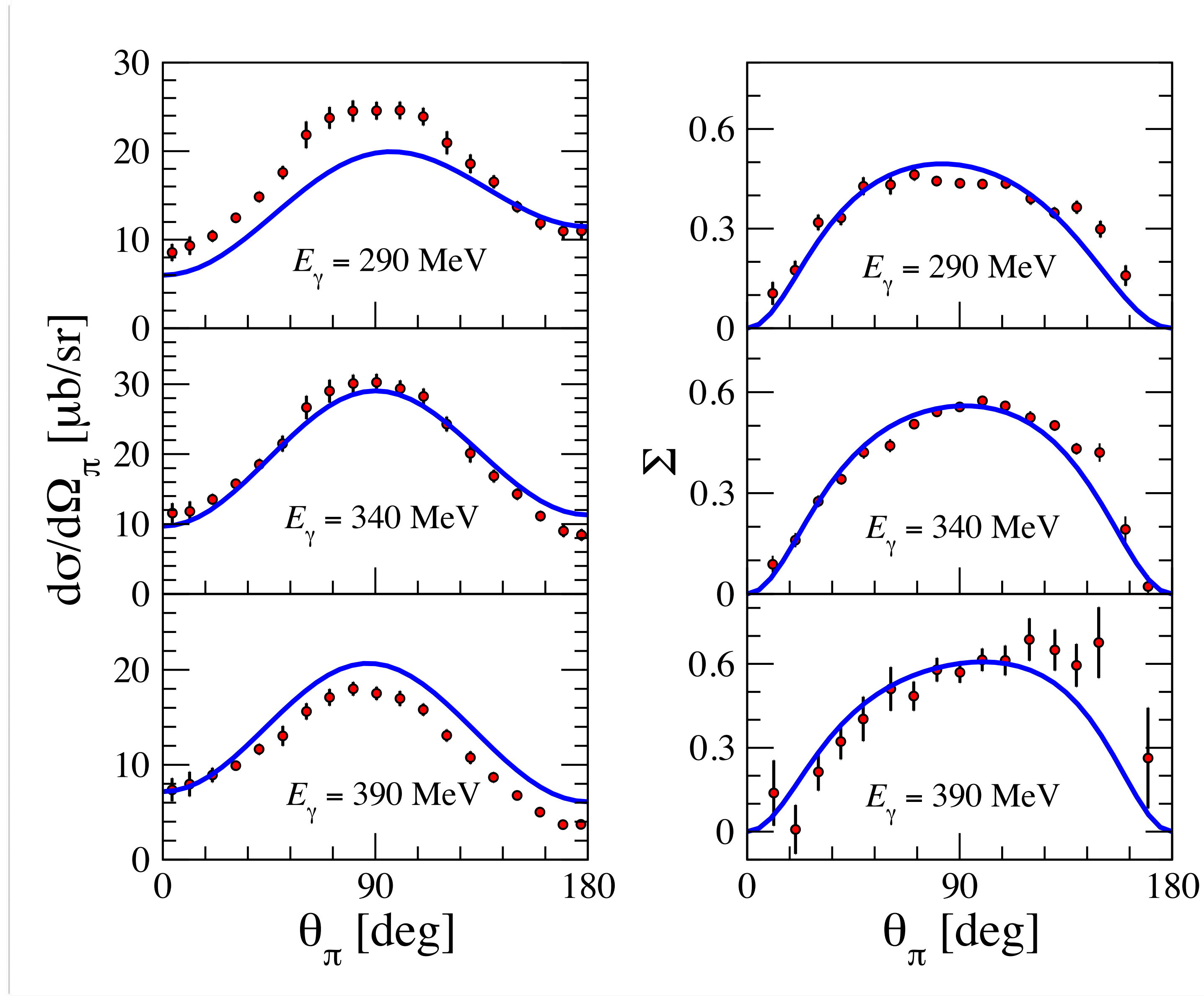


chiral loop corrections: exact unitarity & e.m. gauge-invariance

4 free parameters – LECs corresponding to G_M, G_E, G_C at Q^2=0, and G_M radius.
Only 2 free parameters for photoproduction!

Results for pion photoproduction



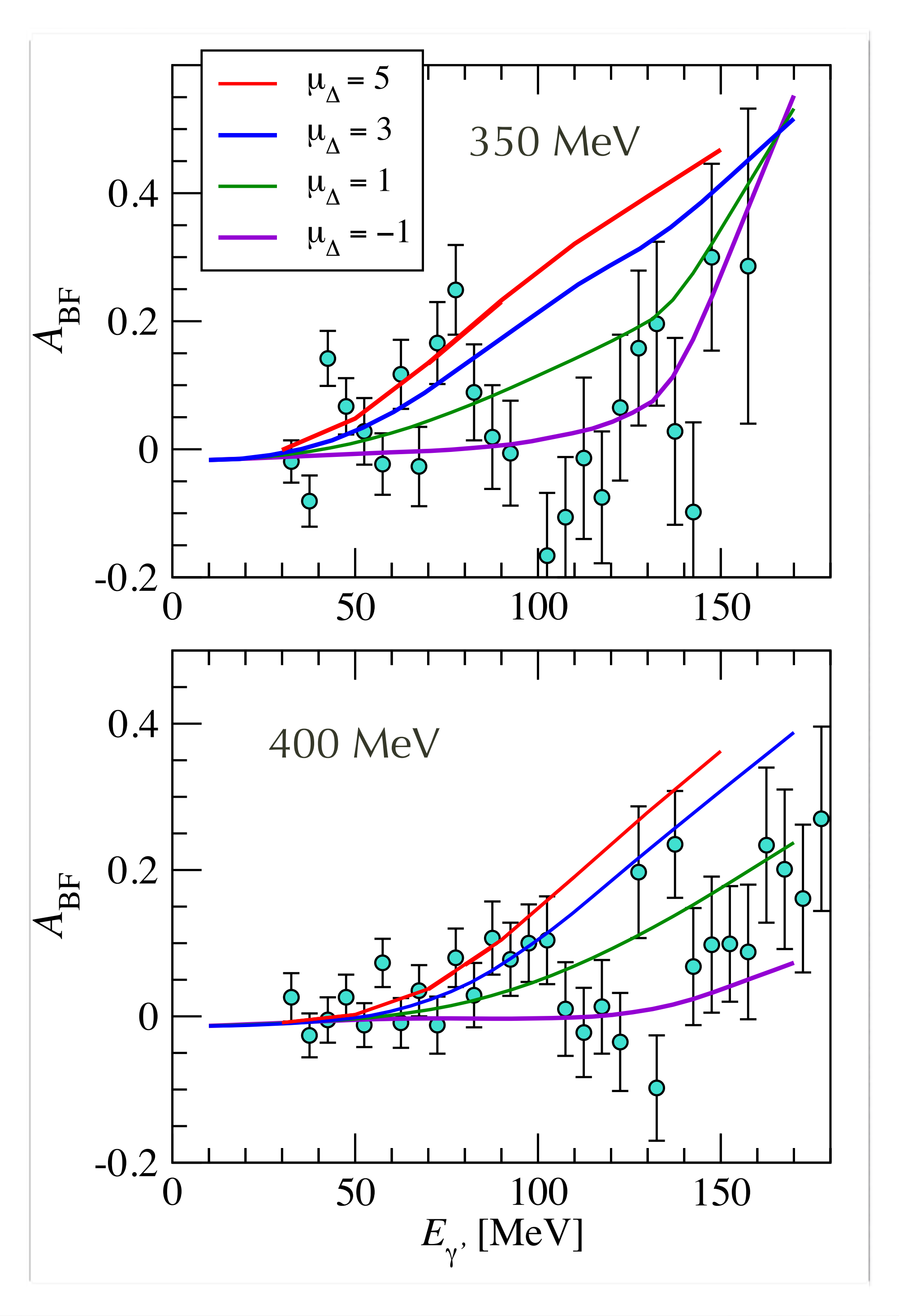


Data: Arends et al (A2 Coll.)
Curves: NLO EFT

Radiative pion photoproduction: BF asymmetry

1) Backward-forward asymmetry:

Divide the Ball into F and B hemispheres, add events where outgoing pion and gamma′ land in the SAME hemisphere with -, in the OPPOSITE with +.
Divide by the total.



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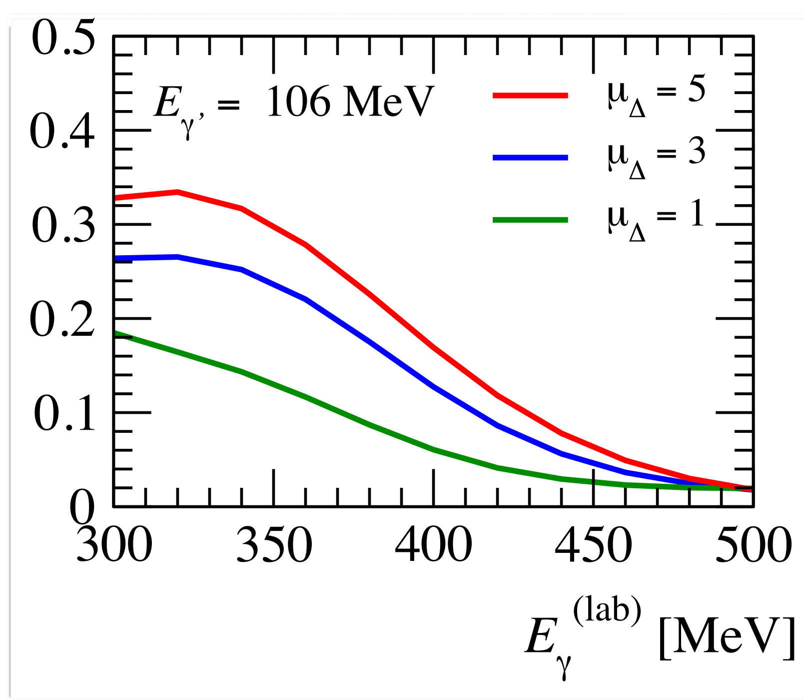

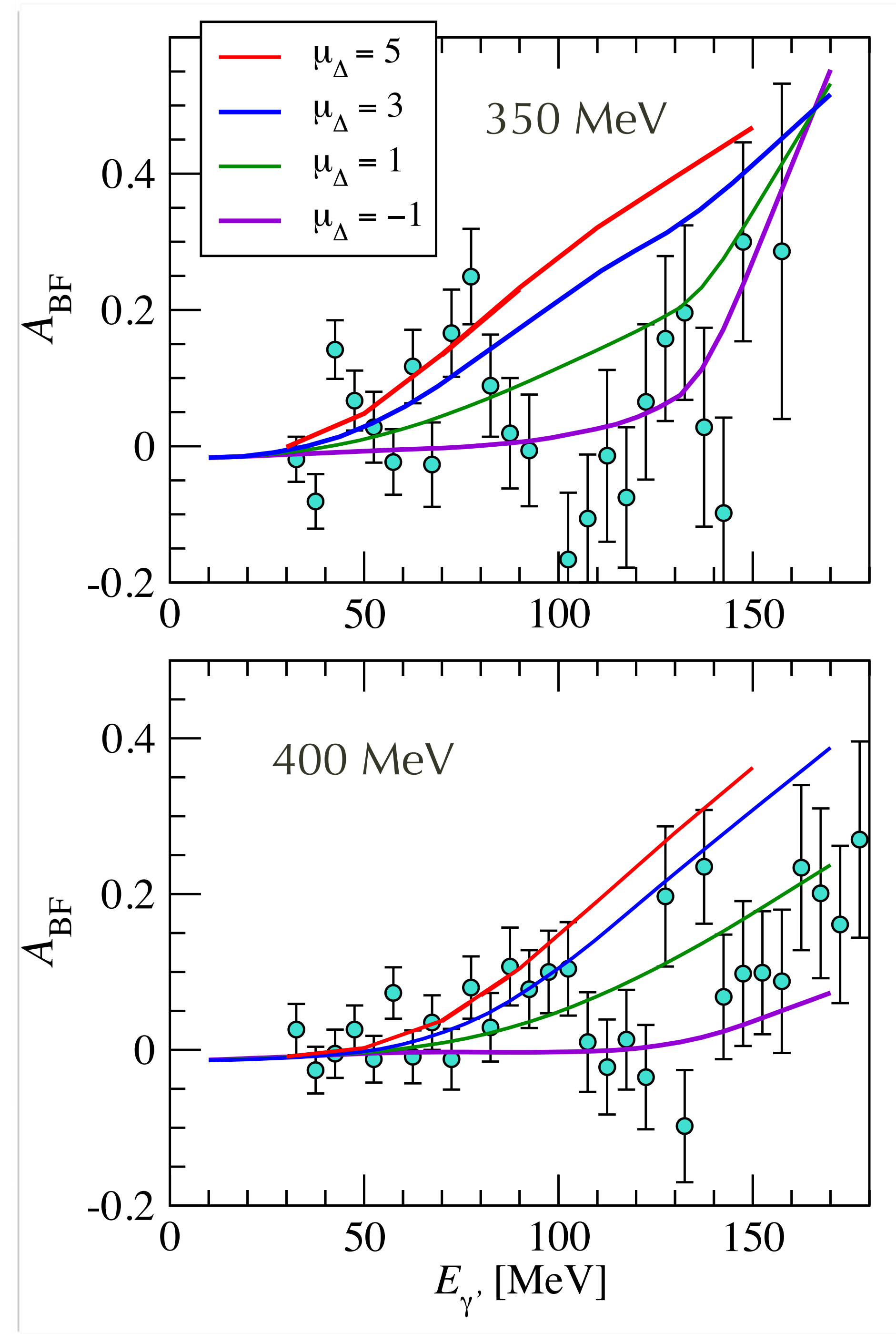

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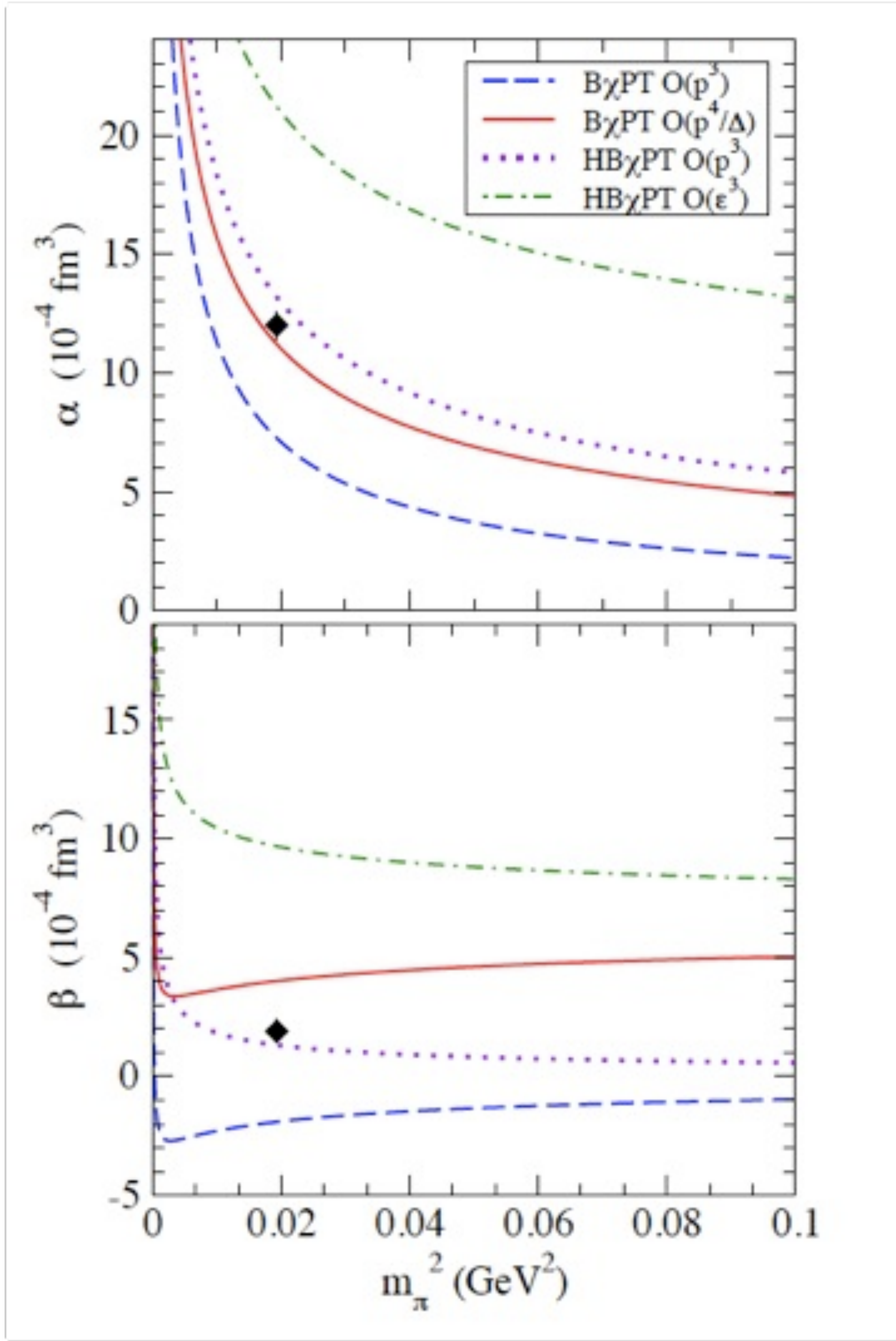
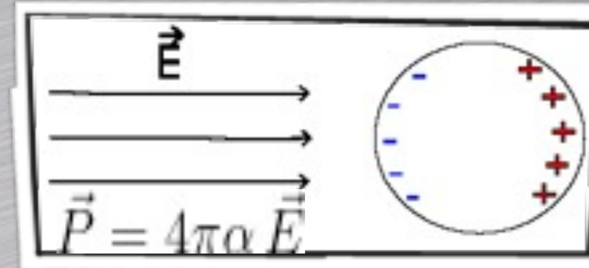
Chiral expansion: HBChPT vs BChPT





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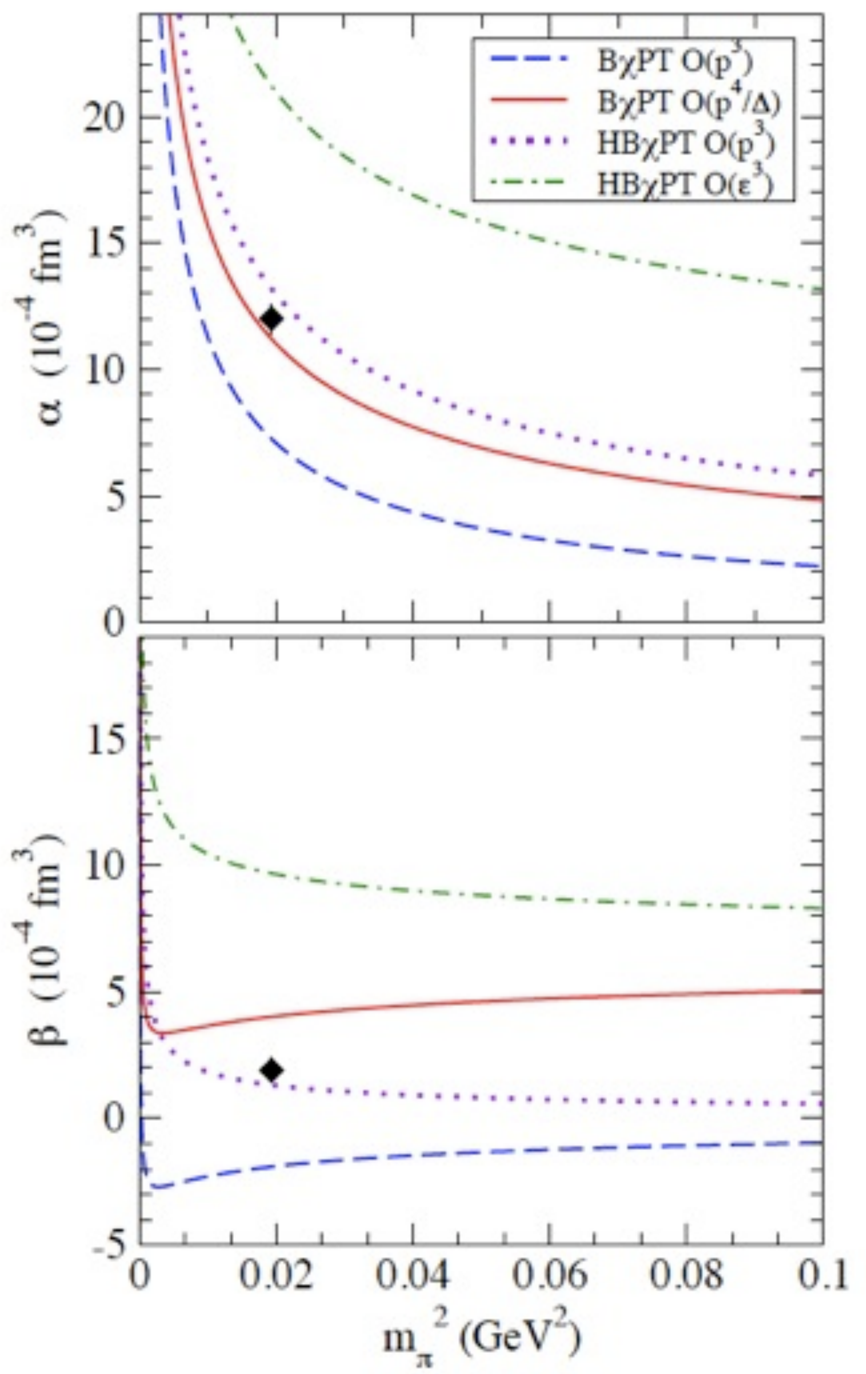
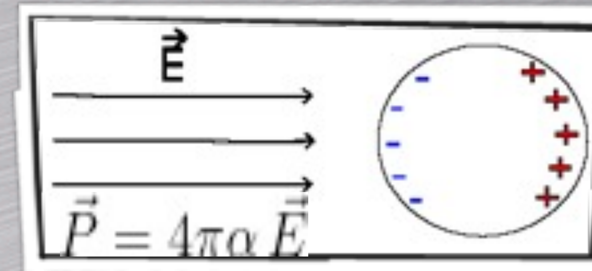




$$\mu = m_\pi / M_N$$

$$(\bar\alpha + \bar\beta)_n = \frac{e^2 g^2}{(4\pi)^2 M^3}\frac{11}{48\mu}\left(1 + \frac{4(1+12\ln\mu)}{11\pi}\mu - \frac{117}{88}\mu^2 + \ldots\right)$$

$$(\bar\alpha + \bar\beta)_p = \frac{e^2 g^2}{(4\pi)^2 M^3}\frac{11}{48\mu}\left(1 + \frac{48(4+3\ln\mu)}{11\pi}\mu - \frac{1521}{88}\mu^2 + \ldots\right)$$

or, numerically,

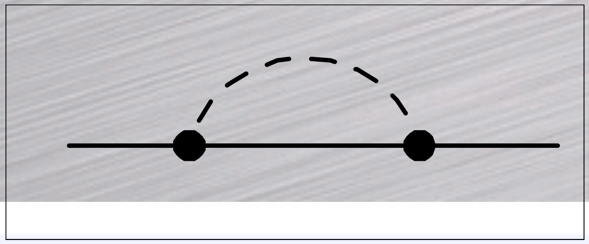
$$(\bar\alpha + \bar\beta)_n = 14.5 - 5.5 - 0.4 + \ldots = 8.7$$

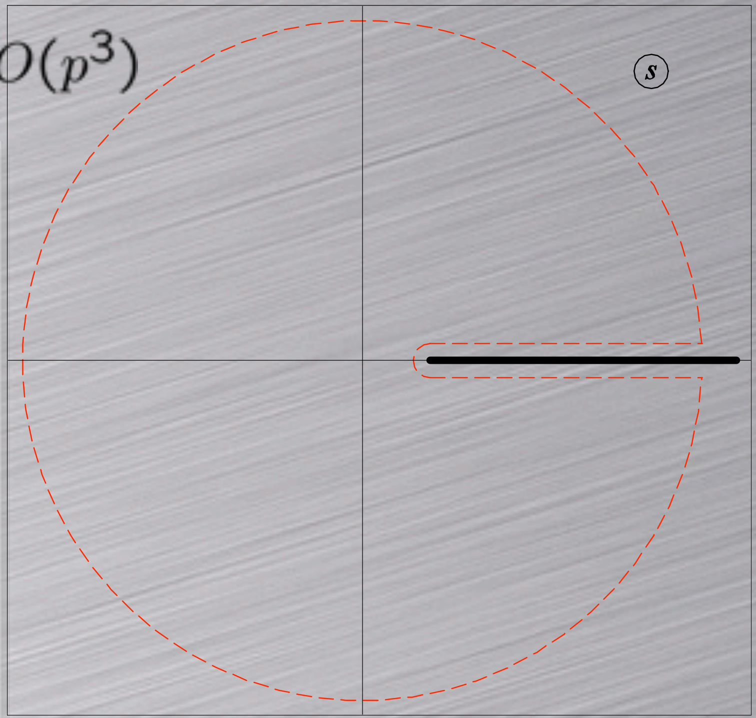
$$(\bar\alpha + \bar\beta)_p = 14.5 - 5.2 - 5.5 + \ldots = 5.3$$

in units of 10^{-4} fm^3.

$$\beta = \frac{e^2 g_A^2}{192\pi^3 F^2 M_N}\left[\frac{\pi}{4\mu} + 18\log\mu + \frac{63}{2} - \frac{981\pi\mu}{32} - (100\log\mu + \frac{121}{6})\mu^2 + \mathcal{O}(\mu^3)\right]$$

Complex-energy vs complex-mass planes

LO nucleon self-energy =  $\equiv \Pi^{(3)}(s, m_\pi^2) = O(p^3)$



Dispersion in energy:

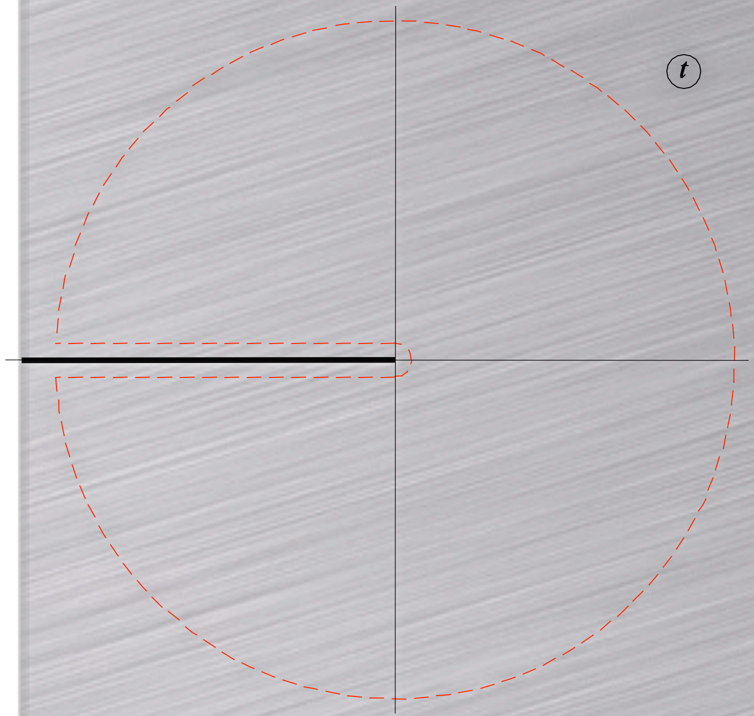
$$\Pi(s, m_\pi^2) = \frac{1}{2\pi i}\oint ds' \frac{\Pi(s', m_\pi^2)}{s' - s}$$

$$\Pi^{(n)}(s, m_\pi^2) = \frac{1}{\pi}\int_{M^2}^{\infty} ds' \frac{\mathrm{Im}\,\Pi^{(n)}(s', m_\pi^2)}{s' - s}\left(\frac{s - M^2}{s' - M^2}\right)^{n-1}$$

Dispersion in $t = m_\pi^2 \sim m_q$:

$$\Pi(M^2, t) = \frac{1}{2\pi i}\oint dt' \frac{\Pi(M^2, t')}{t' - t}$$

$$\Pi^{(n)}(M^2, t) = \frac{1}{\pi}\int_{-\infty}^{0} dt' \frac{\mathrm{Im}\,\Pi^{(n)}(t')}{t' - t}\left(\frac{t}{t'}\right)^{n-1}$$



[Ledwig, V.P., Vanderhaeghen, Phys. Lett. B (2010)]

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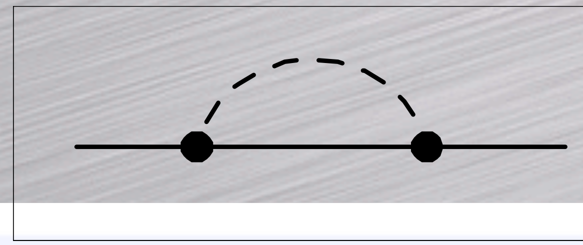
LO nucleon self-energy =  $\equiv \Pi^{(3)}(s, m_\pi^2) = O(p^3)$

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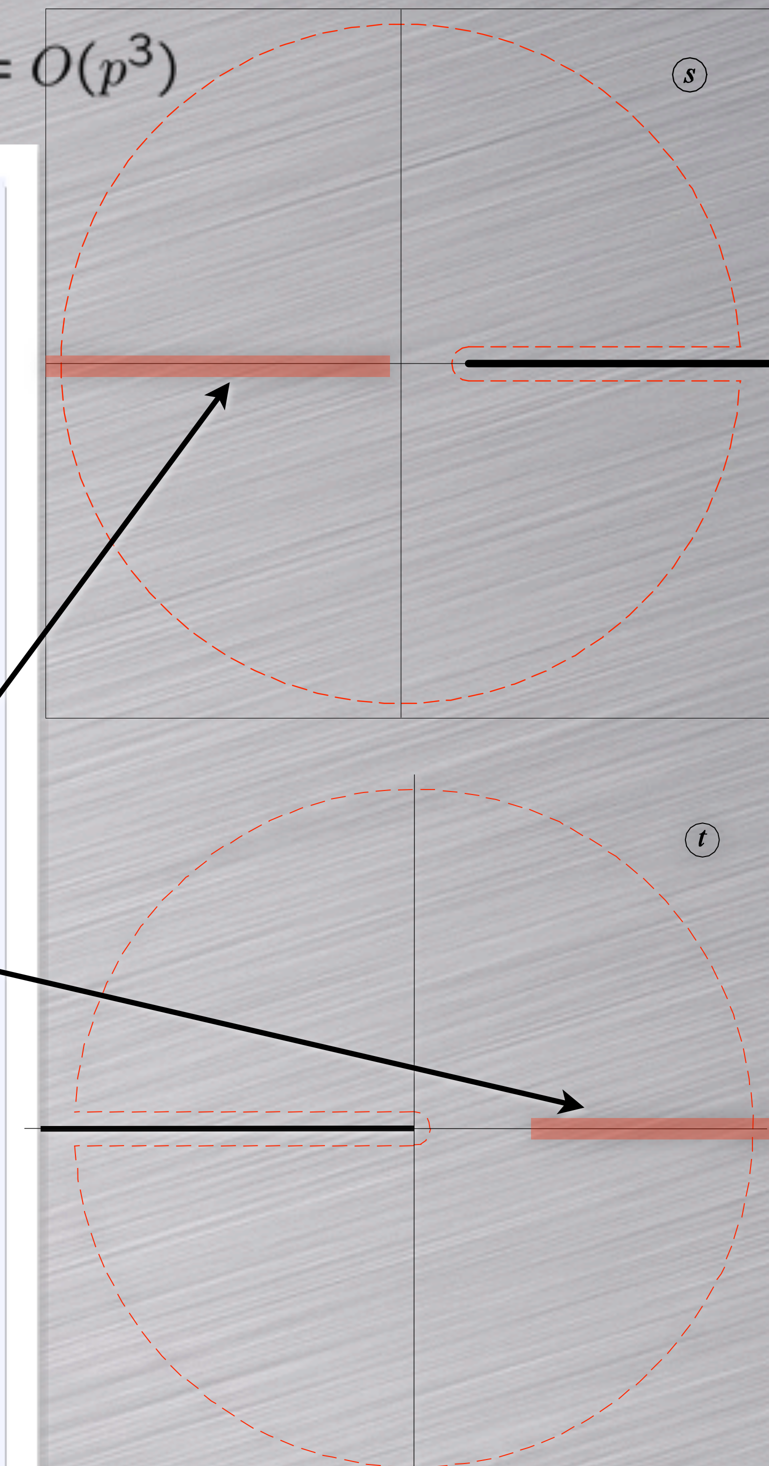
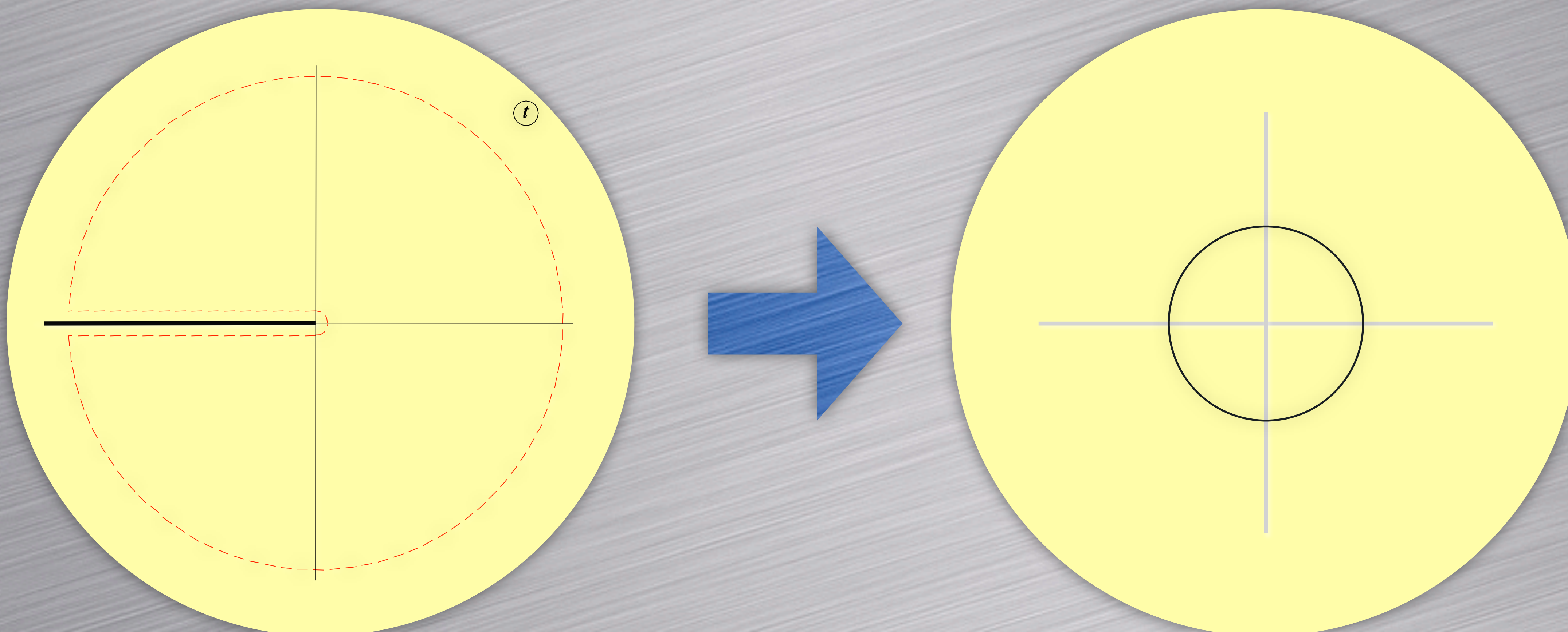
[Ledwig, V.P., Vanderhaeghen, Phys. Lett. B (2010)]



IR cuts

Conformal mapping and analytic continuation



Convergence of a series is limited by the cut at t=0, one can increase the radius of convergence by conformally mapping t=plane such that the cut maps onto the unit semicircle

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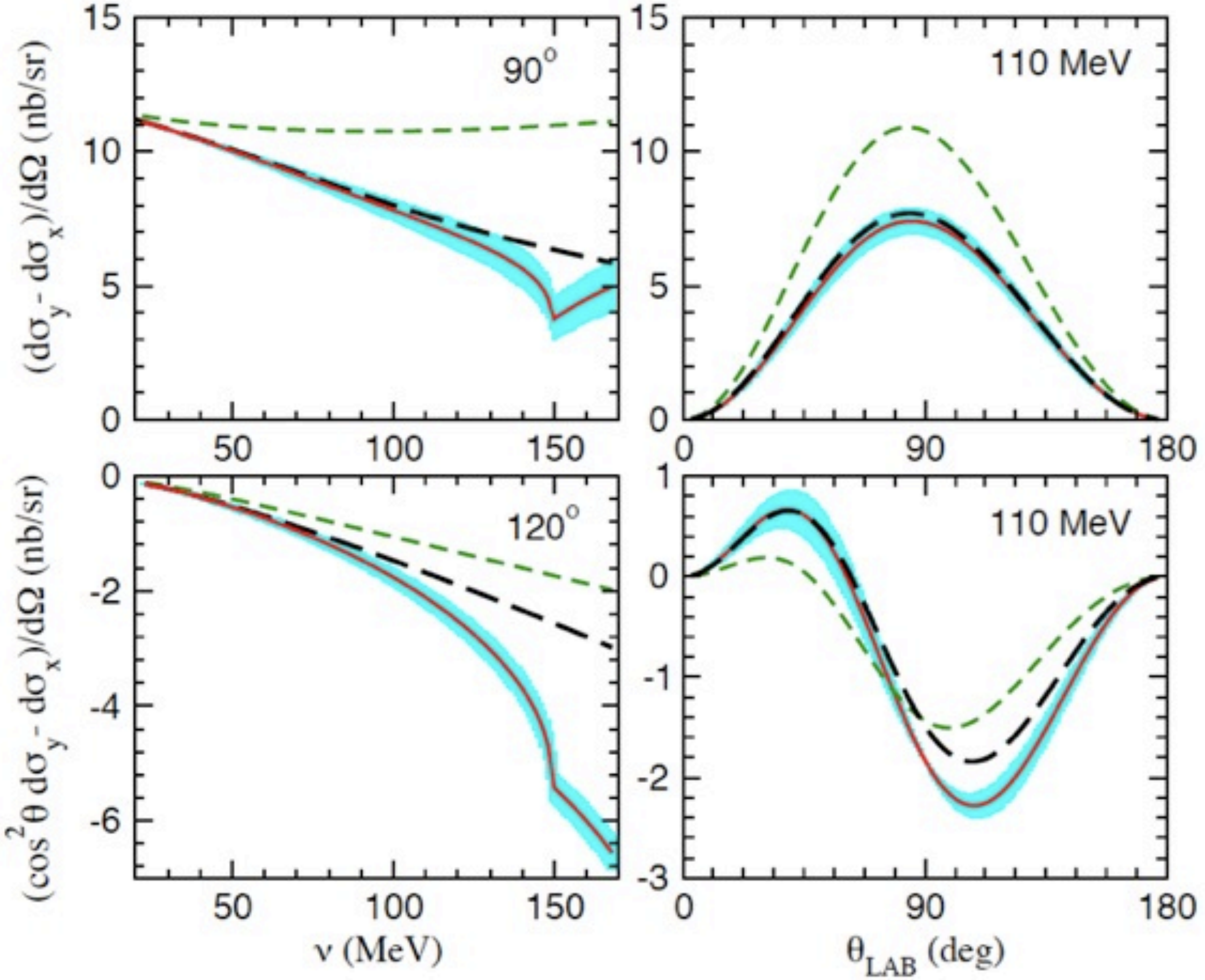
- Delta's MDM @ MAMI can possibly be determined using the new BF asymmetry
- Analytic structure of pion-mass dependence (at least in ChPT) is simple, and can be used to achieve technical advantages and insight into the convergence problem.

Polarized observables (HIGS proposal)



$\alpha^{(p)}$

$\beta^{(p)}$

Born + WZW

LEX

BChPT